

Constraint Supersymmetry Breaking and Non-Perturbative Effects in String Theory¹

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We discuss supersymmetry breaking mechanisms at the level of low energy $\mathcal{N} = 1$ effective heterotic superstring actions that exhibit $SL(2, Z)_T$ target space modular duality or $SL(2, Z)_S$ strong-weak coupling duality. The allowed superpotential forms use the assumption that the source of non-perturbative effects is not specified and as a result represent the most general parametrization of non-perturbative effects. The minimum values of the limits on the parameters in the superpotential may correspond to vacua with vanishing cosmological constant.

One of the biggest problems that heterotic string theory, and its "equivalents", e.g type II, I, have to face today is the question of four dimensional $\mathcal{N} = 1$ space-time supersymmetry breaking. The breaking, due to the presence of the gravitino, that determines the scale of supersymmetry breaking, in the effective action, must be spontaneous and not explicit. Several mechanisms have been used in recent years to break consistently supersymmetry. They can be distinguished as to when they are at work at the string theory level or at the effective superstring action level. The first category of mechanisms includes the tree level coordinate dependent compactification mechanism [1], the magnetized tori approach [2], the type I brane breaking [3, 4, 5], the partial breaking [6] while the latter category includes approaches that use target space duality e.g [7, 8, 9, 10] or S-duality [11, 12] at the level of effective superstring action, related to gaugino condensation [13], to constrain the allowed superpotential forms. The main problem in all of the approaches is the creation of an appropriate potential for the moduli and the dilaton that can fix their vacuum expectation values. Because heterotic string theory has target space duality as one of its properties we can use modular forms to parametrize the unknown non-perturbative dynamics [10], practically to parametrize the unknown non-perturbative contributions to the gauge kinetic function f .

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The purpose of this paper is to reexamine the issue of constructing superpotentials W that affect supersymmetry breaking at the level of $\mathcal{N} = 1$ effective heterotic superstring actions when the source of non-perturbative effects, is not specified.

The modification of the $\mathcal{N} = 1$ heterotic effective action that we examine in this work amounts to modifying the superpotential when T -duality or S -duality non-perturbative effects are included. Moreover we want to break supersymmetry dynamically, rather than geometrically, thus we make use of gaugino condensation in supergravity [14].

In string theories physical quantities like masses of matter, Higgs fields depend on the moduli fields Φ . The latter fields have flat potential to all orders of string perturbation theory, so their vacuum expectation values remains undetermined.

In general there are two different approaches in describing gaugino condensation. These are the effective lagrangian approach [7, 9], where we can use a gauge singlet bilinear superfield U as a dynamical degree of freedom [14, 15] and the effective superpotential approach [8, 10, 16, 17, 18]. In the latter formalism the gauge singlet bilinear superfield is integrated out through its equation of motion.

The $\mathcal{N} = 1$ effective supergravity theory coming from superstrings is described by the knowledge of three functions, the Kähler potential K , the superpotential W and the gauge kinetic function f , that all depend on the moduli fields.

In general duality stabilizes the potentials with local minima at the points $T, S = 1, \rho$. The minima in the cases considered in [11, 10] are either at the self-dual points giving unbroken space-time supersymmetry in the T, S field sector or in the general case supersymmetry breaking minima with negative cosmological constant. In the latter case the minima occur at the boundary of the moduli space.

While considerable progress have been made to the understanding of non-perturbative effects based on D-branes, the problem of determining the moduli values still remains. In the absence of such a non-perturbative mechanism, we choose to break $\mathcal{N} = 1$ local SUSY, via gaugino condensation. We choose a four dimensional heterotic orbifold [19] with Kähler potential $K = -3 \log(T + \bar{T})$, where T the complex structure modulus.

For $(2, 2)$ heterotic string compactifications with $\mathcal{N} = 1$ supersymmetry there is at least one complex modulus T which we denote by $T = R^2 + ib$, where R is the breathing mode of the six dimensional internal space and b is the internal axion θ . The T -field corresponds, when T large, to the globally defined $(1, 1)$ Kähler form. Here we will restrict our study to the simplest $(2, 2)$ models where there is a single overall modulus, by freezing all other T -moduli.

The four dimensional string effective supergravity action is invariant to all orders of perturbation theory under the target space duality transformations

$$T \rightarrow \frac{AT - iB}{iCT + D}, \quad AD - BC = 1, \quad (0.1)$$

since $G = K + \log |W|^2$ has to remain invariant, while the superpotential transforms with modular weight -3 , $W \rightarrow (iCT + D)^{-3}W$. Local supersymmetry is spontaneously broken if the auxiliary fields $h^i = W^{(K/2)}(K^i + W^i/W)$ gets non-vanishing vacuum expectation values. If we choose the superpotential W in the, factorized, form $W(T, S) = \Omega(T)K(S)\eta^{-6}$, $K(S) = e^{-3S/2b}$ that is equivalent to defining the gauge kinetic function into the form

$$f = S - \frac{|G_i|}{|G|} (b_a^{N=2} \log[\eta^4(T)(T + \bar{T})] - \frac{1}{16\pi^2} \text{Re}\{\partial_T \partial_U h^{(1)}(T, U) - 2 \log((j(T) - j(U)))\}) + (b_a/3) \log |\Omega(T)|^2 + \mathcal{O}(e^{-S}), \quad (0.2)$$

where $h^{(1)}$ is the one-loop prepotential², G_i the orders of the subgroup G which leaves the i -complex plane unrotated and we have included the one-loop Green-Schwarz term. Because $\Omega(T)$ parametrizes our ignorance of non-perturbative corrections (from target space duality) for the T -modulus, it was assumed [10] that the T -modulus dependent part of the superpotential takes the form

$$W(T) = (j - 1728)^{m/2} j^{n/3} [\eta(T)]^{2r} \mathcal{P}(j(T)), \quad (0.3)$$

$$W(T) = \Omega(T) [\eta(T)]^{2r} \mathcal{P}(j(T)), \quad (0.4)$$

$$\Omega(T) = (j - 1728)^{m/2} j^{n/3}, \quad (0.5)$$

where m, n positive integers and G_6, G_4 are the Eisenstein functions of modular weight six and four and $\mathcal{P}(j(T))$ an arbitrary polynomial of the absolutely $SL(2, Z)$ modular invariant $j(T)$. Analyzing the supergravity scalar potential corresponding to (0.5) we found negative cosmological constant at its minimum. The minima on the cases considered in [11, 10] are either at the self-dual points giving unbroken space-time supersymmetry in the T, S field sector respectively or in the general case supersymmetry breaking minima with negative cosmological constant. In the latter case the minima occur at the boundary of the moduli space.

In addition, depending on the value of $(S + \bar{S})\partial K(S)/\partial S$ the dilaton minimum can be at weak or strong coupling [10]. However, because W is a section on a flat holomorphic bundle over the moduli space, the constraints

$$n \bmod 3, m \bmod 2, \quad (0.6)$$

that have to be included apriori [12] in (0.5) under the dilatational transformations $T \rightarrow T + 1$ are missing. The construction of the superpotentials that incorporates apriori the constraints (0.7) is performed in [23] and treats not the η -invariant as a fundamental quantity, as in (0.5),

²The one loop $\mathcal{N} = 2$ four dimensional vector multiplet prepotential $h^{(1)}$ was calculated as an ansatz solution to a differential equation involving one loop corrections to gauge coupling constant in [20]. However, its exact general form for any four dimensional compactification of the heterotic string was calculated in [21]. Higher derivatives of $h^{(1)}$ were calculated in [22].

but rather the cusp forms $\Delta(T)$ instead. As a result, by writing down for a moduli field Φ the most general weight zero modular form

$$j^a(\Phi)(j - 1728)^b(\Phi), \quad (0.7)$$

where a, b integers and j is the $SL(2, Z)$ modular function, (0.5) is generalized³ and the following new T -modulus superpotentials are both allowed, namely

$$W(T) = \frac{\tilde{\Sigma}(T)}{\eta^6(T)} \mathcal{P}(j), \quad (0.8)$$

$$W(T) = \frac{1}{\eta^6(T)} \frac{1}{\tilde{\Sigma}(T)} \mathcal{P}(j). \quad (0.9)$$

Defining the candidate T -dual superpotentials into the forms (0.8), (0.9) is equivalent to demanding that we do not allow or do allow poles in the upper half plane respectively. This happens because modular functions which are allowed to have poles in upper half plane are exactly rational functions in j (quotients of polynomials in j) whereas modular functions which are not allowed to have such poles are polynomials in j . Including the dilaton part $K(S)$ in (0.8), (0.9) results in the construction of the full superpotential. The superpotentials (0.8), (0.9) can be further constrained by examining stability constraints on the scalar potential at the self-dual points $T = 1$, $T = \rho = e^{i\pi/6}$. By employing the latter method we derive minimum values on the a, b parameters,

$$(a; b) = (1, 2, 3, \dots; 2, 3, \dots). \quad (0.10)$$

It is worth noticing that the stable minimum values of the a, b parameters correspond to vacua with vanishing cosmological constant at a generic point in the moduli space. The new non-perturbative superpotentials may be used to test the nature of non-perturbative effects in the 4D orbifold constructions of the heterotic string.

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³see for example [24]

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